MOTION OF LARGE GAS BUBBLES ASCENDING IN A LIQUID

I. G. Malenkov

Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, Vol. 9, No. 6, pp. 130–134, 1968

An equation is derived for the ascent velocity of large gas bubbles in a liquid. This velocity is assumed to be governed by the propagation of a wavelike perturbation caused by the bubble in the liquid.

NOTATION

w—bubble (or drop) velocity
γ—specific gravity
μ—dynamic viscosity
ν—kinematic viscosity
r—bubble (or drop) radius
σ—surface tension
ξ—coefficient of friction
g—gravitational acceleration
D—bubble (or drop) diameter
p—pressure
c—propagation velocity of the wavelike perturbation
λ—wavelength

A single prime indicates the heavy phase of the system, two primes indicate the light phase, and the subscript m indicates extreme values.

There have been many theoretical [1–3] and experimental [4–6] studies of the ascent velocity of bubbles in liquids, but the relationship between this velocity and the bubble dimensions has not yet been definitely established. At present, there are at least four different regions in the bubble-diameter range from 0 to 20 mm requiring different calculations procedures [7].

1. **Region of laminar flow around bubbles retaining spherical shape.** This region is limited to Reynolds numbers R < 2. The ascent velocity is given by

$$w = \frac{2}{9}\frac{\gamma' - \gamma''}{\mu'} r^2. \tag{1}$$

2. **Region of the motion of bubbles in the shape of planar, pulsating spheroids.** This region is limited to Reynolds numbers within the range $2 < R < 4A^{0.42}$. The equation recommended for determining the bubble ascent velocity is

$$w = 0.33\frac{g^{0.76}}{\nu^{0.52}} r^{1.28}. \tag{2}$$

3. **Region of the motion of planar, relatively stable bubbles.** This region is restricted to the Reynolds number range $4A^{0.42} < R < 3A^{0.5}$. The equation for the ascent velocity is

$$w = 1.35\left(\frac{g\sigma}{r(\gamma' - \gamma'')}\right)^{0.5}. \tag{3}$$

4. **Region of the motion of mushroom-shaped bubbles.** This region is limited to Reynolds numbers $R > 3A^{0.5}$. In this extremely broad region, the bubble velocity is calculated from

$$w = \left(\frac{4g^2\sigma(\gamma' - \gamma'')}{\xi^2\gamma'^2}\right)^{1/4}. \tag{4}$$

The quantity A, which defines the limits of applicability of Eqs. (1)–(4), is given by

$$A = \frac{\sigma^{3/2}\gamma'}{g\mu'^2\sqrt{\gamma' - \gamma''}}.$$

Figure 1 shows experimental data from [6, 8, 9] showing the dependence of the ascent velocity of individual air bubbles in water at $p = 9.8 \cdot 10^4$ N/m². The dependence is seen to be quite complicated. However, the division of the entire D range into four characteristic regions cannot be considered to be a natural one, reflecting some real difference in the mechanisms governing the bubble velocity in the given medium. The artificiality of the definition of these regions is also evident from the structure of Eqs. (1)–(4).

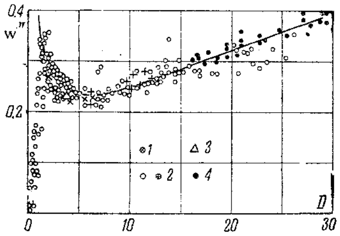

Fig. 1. Dependence of the ascent velocity w'' (m/sec) on the bubble diameter D(mm) at $p = 9.8 \cdot 10^4$ N/m². 1-4) data from [6, 8, 9, 15, respectively].

We propose here a "universal" equation, i. e., one correlating the experimental data in the third and fourth regions. This equation was obtained under the assumption that the bubble ascent velocity is identically equal to the propagation phase velocity of capillary waves of length πD in the liquid. This assumption can be easily understood on the basis of the following arguments.

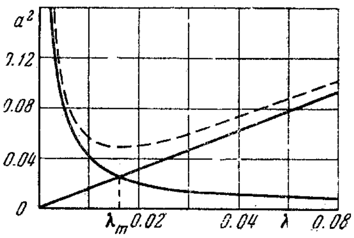

Fig. 2. Dependence of the propagation velocities a^2 (m/sec²) of capillary and gravitational waves on the wavelength λ(m).

The small value of the friction between a liquid in gas at a small relative velocity is due to the nondetached nature of the flow around the ascending bubbles [10]. This factor also tends to improve the elasticity of the interface and the small "rigidity" of large bubbles, since any local change in pressure causes a corresponding change in shape. Accordingly, it may be assumed that the liquid particles flowing around a bubble are not caught up in the bubble motion, but instead, remaining at the same horizontal level, merely undergo oscillations about an equilibrium position. The oscillation velocity and the propagation velocity of a wavelike perturbation are related by

$$v = \Delta p / \rho' a. \tag{5}$$

Here Δp is the excess pressure in the wave, $\rho' a$ is the characteristic impedence in the medium, ρ' is the density of the medium, and a is the wave propagation velocity.

The mechanical energy of an oscillating system consists of the kinetic and potential energies. For a no-loss system, periodic changes of one type of energy into the other are characteristic; at any time, the total oscillatory energy per unit volume is, according to [11],

$$E = {}^1\!/_2\, \rho' v^2. \tag{6}$$

The kinetic energy is carried by the mass element, while the potential energy is carried by the elastic element. In this system, the elastic element is characterized by the surface tension and the area of the interface. Therefore, it is obvious that

$$E=\sigma\left(\frac{1}{r_1}+\frac{1}{r_2}\right). \tag{7}$$

The horizontal radius r_1 of the bubble does not play an important role, since the forces acting in this plane compensate for each other (it is assumed that the bubble has a nearly circular cross section in this plane). Relation (7) may therefore be rewritten in terms of the vertical radius alone:

$$E \approx \sigma/r_2 . \tag{8}$$

Comparing (6) and (8), we find the oscillatory velocity of the system to be

$$v=\sqrt{2\sigma/r_2\rho'} . \tag{9}$$

The excess pressure in this oscillatory system is a Laplace correction which is generally equal to (7). Experiments yield $r_1 = (1.15-1.20)r_2$ for relatively small bubbles ($D \approx 10$ mm). Therefore, within an error of 15–20%, we may assume

$$\Delta p = 2\sigma/r_2 . \tag{10}$$

As noted below, an increase in bubble size reduces the error.

Setting $r_2 = r$, and substituting (9) and (10) into (5), we find the propagation velocity for a wavelike perturbation in the liquid to be

$$a=\sqrt{2\sigma/r\rho'} . \tag{11}$$

The quantity r is given in order of magnitude by

$$r \sim \sqrt{\sigma/(\gamma'-\gamma'')} ;$$

substituting this into (11), we find

$$a \approx \left(\frac{4g^2\sigma(\gamma'-\gamma'')}{\gamma'^2}\right)^{1/4} . \tag{12}$$

Comparing (12) and (4), we see that when $\xi = 1$, the velocities w and a are equal; i. e., the wave propagation velocity is equal to the bubble ascent velocity. Under which conditions $\xi = 1$ will actually hold in (4) is shown below.

Accordingly, it turns out that the motion of the liquid particles as they flow around the bubbles is actually similar to that which occurs in the case of a wavelike motion of the interface between two immiscible liquids, and that the velocity of an object carrying the perturbation in the liquid is equal to the propagation velocity of a wave formed at the interface.

It is known from the theory of surface waves that there are two limiting types of waves, gravitational and capillary, which are separated by an intermediate range of wavelengths in which the motion is of a mixed nature. Figure 2 shows the dependences of the velocity of gravitational (straight line) and capillary (hyperbolic curve) waves on their wavelength. The combined effects of gravitational and capillary forces on the wave velocity are shown by the dashed curve. In the general case (dashed curve), the propagation phase velocity of the wavelike perturbation is described by [11]

$$a^2=\frac{g\lambda}{2\pi}\,\frac{\gamma'-\gamma''}{\gamma'+\gamma''}+\frac{2\pi}{\lambda}\,\frac{g\sigma}{\gamma'+\gamma''}, \tag{13}$$

where λ is the wavelength. The minimum wave velocity corresponds to the wavelength

$$\lambda_m=2\pi\sqrt{\sigma/(\gamma'-\gamma'')} . \tag{14}$$

A comparison of the experimental curve in Fig. 1 with the resultant (dashed) curve in Fig. 2 shows that, as expected on the basis of the above arguments, there is a quantitative as well as a qualitative similarity.

Accordingly, in order that Eq. (13) be suitable for calculating the bubble ascent velocity, it is sufficient to establish the relation between the wavelength λ with the dimensions of the bubble exciting this wave. Since we have found that the bubble ascent velocity and the wave propagation velocity are equal, this is easily done. We assume that the minimum bubble ascent velocity (Fig. 1) corresponds to the minimum resultant wave propagation velocity (Fig. 2).

The minimum wavelength λ_m is easily calculated from Eq. (14). For water, e.g., we have $\lambda_m = 17.1$ mm at 20° C. It is evident from Fig. 1 that $D_m \approx 5.5$ mm. Accordingly, we have

$$\lambda_m / D_m = \pi. \tag{15}$$

This relation should evidently hold for all liquids.

Since $\lambda = \lambda_m$ is a particular case of the possible wavelength values for which the velocity is given by Eq. (13), it may be assumed that Eq. (15) will also hold for the entire wavelength range; i.e.,

$$\lambda = \pi D. \tag{16}$$

Substitution of (16) into (13) yields

$$w = \left(\frac{gD}{2} \frac{\gamma' - \gamma''}{\gamma' + \gamma''} + \frac{2g\sigma}{D(\gamma' + \gamma'')} \right)^{1/2}. \tag{17}$$

The solid curve in Fig. 1 shows data calculated from Eq. (17). This equation describes the experimental data completely satisfactorily over a wide range of bubble diameters $D \gtrsim 1.5$ mm.

The expression in the radical in Eq. (17) is the sum of gravitational (first term) and capillary (second term) forces. As D increases, the role of the second term decreases, so the error introduced by the inaccuracy of Eq. (16) also decreases.

Interestingly, when $\lambda = \lambda_m$ Eq. (13) yields

$$w = \left(\frac{4g^2\sigma(\gamma' - \gamma'')}{(\gamma' + \gamma'')^2} \right)^{1/4}; \tag{18}$$

at pressures close to atmospheric, i.e., when the quantity γ'' can be neglected, this relation is analogous to Eq. (4), obtained by Frank-Kamenetskii for the ascent velocity of large, deformed bubbles [7]. Accordingly, Eq. (4) is a particular case of the more general relation (17) and holds in an extremely narrow region of bubble sizes, near the value $D = \lambda_m/\pi$ (in the range 4 mm < D < 7 mm), where the ascent velocity actually depends weakly on the bubble size. Only in this case can the drag coefficient in (4) be assumed equal to unity. The introduction of the quantity ξ into (4) should be considered as being necessary to take into account unknown factors affecting the bubble ascent velocity.

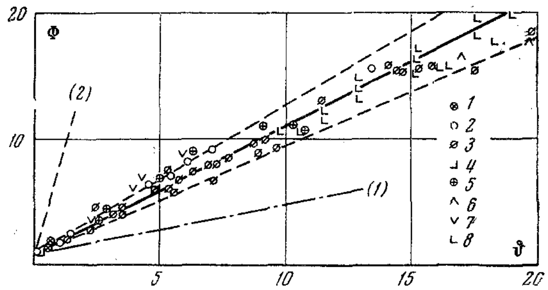

Fig. 3. Comparison of experimental data of various authors on the motion of bubbles and drops in a liquid with data calculated from Eq. (19), in terms of the dimensionless quantities Φ and v: 1–4) water-air [13, 6, 8, 15, respectively]; 5) saponin-air [8]; 6) cyclohexane [8]; 7) water-hydrogen [8]; 8) water-air [9].

Although Eq. (17) was obtained for the ascent velocity of bubbles, there is no fundamental difference between the mechanisms for the ascent of bubbles and drops, so Eq. (17) should also hold for the motion of drops in liquids. For convenience in comparing the experimental data on the ascent of bubbles and drops by means of Eq. (17), we convert the data to dimensionless form and plot the function

$$\Phi = \Phi(v) \quad \left(\Phi = \frac{w^2 D (\gamma' + \gamma'')}{2g\sigma}, \quad v = \frac{D^2(\gamma' - \gamma'')}{4\sigma}\right). \tag{19}$$

Figure 3 shows experimental and calculated data in terms of the coordinates in (19). Most of the experimental data obtained by different authors for gas-liquid systems are seen to correlate satisfactorily with the calculated data (solid line). This is not true of the experimental data of Peebles and Garber [12] (dot-dash line 1), obtained during bubble motion in a tube of relatively small diameter (25 mm). The deviation of these points is evidently due to the effects of the solid walls on the propagation velocity of the wavelike perturbation. Also shown in this figure are experimental data [13] on the motion of liquid drops (dashed line 2). The qualitative nature of the dependence is retained, but there is a significant quantitative deviation. The actual drop velocities turn out to be lower than those calculated by Eq. (17). The discrepancy between the experimental $w = f(D)$ dependences for drops and bubbles is evidently due to the fact that the liquid flow around a drop is not nondetached, and the interaction between the components at the liquid-liquid interface cannot be neglected. The presence of tangential stresses at the interface causes irreversible energy losses, so the drop moves more slowly.

REFERENCES

1. M. W. Rybczynski, "O ruchy postepowym kuliciektej w osrodku lepkim," "Über die fortschreitende Bewegung einer flüssigen Kudel in einem Sähen Medium," Bull. Internat. Acad. Polon. Sci et Lettr., ser. A, Sci., Math., no. 1 A, 1911.
2. N. T. Hadamard, "Mecanique mouvement permanent lent d'un sphere liquide et visqueuse dans an liquide visqueux," C. R. Acad. Sci., vol. 152, no. 1, 1911.
3. V. G. Levich, Physicochemical Hydrodynamics [in Russian], Izd. AN SSSR, 1952.
4. A. N. Frumkin and I. A. Bagotskaya, "Descent velocity of a mercury drop in a viscous medium," Dokl. AN SSSR, vol. 55, 135–140, 1947.
5. A. V. Gorodetskaya, "Ascent velocity of bubbles in water and aqueous solutions at high Re numbers," Zh. fiz. khimii, vol. 23, no. 1, 1949.
6. B. K. Kozlov and M. A. Mologin, "Ascent velocity and drag of air bubbles in a liquid," Izd. AN SSSR, no. 8, 1951.
7. S. S. Kutateladze and M. A. Styrikovich, Hydraulics of Gas-Liquid Systems [in Russian], Gosenergoizdat, 1958.
8. D. W. Krevelen and P. I. von Hoftizer, "Studies of gas bubble formation. Calculation of interfacial area in bubble contactors," Chem. Engng Progr., vol. 46, no. 1, 1950.
9. R. M. Lodyzhenskii, "Motion of an air bubble in water at high Re numbers," Zh. prikl. khimii, vol. 27, no. 1, 1954.
10. B. B. Brandt and D. I. Perazich, "Flow of a liquid over large gas bubbles," Inzh.-fiz. zh. [Journal of Engineering Physics], vol. 10, no. 2, 1966.
11. H. Lamb, Hydrodynamics [Russian translation], Gostekhizdat, Moscow-Leningrad, 1947.
12. F. N. Peebles and H. I. Garber, "Study of the motion of gas bubbles in liquids," Chem. Engng, vol. 49, no. 2, 1953.
13. N. I. Smirnov and V. L. Ruban, "Relative velocity of drops as a function of the velocity of the medium," Zh. prikl. khimii, vol. 22, no. 10, 1949; "Relative velocity of drops in the transition region," Zh. prikl. khimii, vol. 24, no. 1, 1951.
14. N. I. Smirnov and V. L. Ruban, "Velocity of drops as a function of the velocity of the medium," Zh. prikl. khimii, vol. 22, no. 11, 1949.
15. V. A. Sakharov, "Experimental determination of the relative velocity of a gas bubble in a liquid flow," Izvestiya VUZ. Neft i gaz, no. 6, 1966.

27 May 1968

Novosibirsk